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Mining from Data Streams: Issues and Challenges Part I

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2022

July 2022

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Motivation

- 2 Data Streams
 - Approximate Answers
 - Count-Min Sketch

3 Basic Methods

- Estimating statistics over windows
- Sampling
- Illustrative Applications
 - Toy Problem
 - Hot-Lists
 - Distributed Streams



Data Streams

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Tribute to Sir Ronald Fisher



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Nowadays				

we have machines that collect data, process the information, and send information to other machines ...



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Data Never Sleeps ...



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The New Numbers ...

The Digital Universe: 50-fold Growth from the Beginning of 2010 to the End of 2020



Source: IDC's Digital Universe Study, sponsored by EMC, December 2012

Memory unit	Size	Binary size
kilobyte (kB/KB)	10 ³	2 ¹⁰
megabyte (MB)	10 ⁶	2 ²⁰
gigabyte (GB)	10 ⁹	2 ³⁰
terabyte (TB)	10 ¹²	2 ⁴⁰
petabyte (PB)	10 ¹⁵	2 ⁵⁰
exabyte (EB)	10 ¹⁸	2 ⁶⁰
zettabyte (ZB)	10 ²¹	270
yottabyte (YB)	10 ²⁴	2 ⁸⁰

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The Fourth Industrial Revolution ...



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Internet of Things ...



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 Monthly worldwide Google searches for the application 2. Monthly Tweets containing the application name and #IOT 3. Monthly LinkedIn Posts that include the application name. All methods wald for C42(2014, Sources; Google, Twitter, LinkedIn, IoT Analytics

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Scenario



Electrical power Network: Sensors all around network monitor measurements of interest.

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Scenario

- Sensors produce continuous flow of data at high speed:
 - Send information at different time scales;
 - Act in adversary conditions: they are prone to noise, weather conditions, battery conditions, etc;
- Huge number of Sensors, variable along time
- Geographic distribution:
 - The topology of the network and the position of the sensors are known.

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Illustrative Learning Tasks:

- Cluster Analysis
 - Identification of Profiles: Urban, Rural, Industrial, etc.
- Predictive Analysis
 - Predict the value measured by each sensor for different time horizons.
 - Prediction of peaks on the demand.
- Monitoring Evolution
 - Change Detection
 - Detect changes in the behavior of sensors;
 - Detect Failures and Abnormal Activities;
 - Extreme Values, Anomalies and Outliers Detection
 - Identification of peaks on the demand;
 - Identification of critical points in load evolution;

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Standard Approach:

This problem has been addressed time ago:

Strategy

- Select a finite sample
- Generate a static model (cluster structure, neural nets, Kalman filters, Wavelets, etc)
- Very good performance in next month!
- Six months later: Retrain everything!

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Standard Approach:

This problem has been addressed time ago:

Strategy

- Select a finite sample
- Generate a static model (cluster structure, neural nets, Kalman filters, Wavelets, etc)
- Very good performance in next month!
- Six months later: Retrain everything!

What is the Problem?

The world is not static! Things change over time.

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The Data Stream Phenomenon

- Highly detailed, automatic, rapid data feeds.
 - Radar: meteorological observations.
 - Satellite: geodetics, radiation,.
 - Astronomical surveys: optical, radio,.
 - Internet: traffic logs, user queries, email, financial,
 - Sensor networks: many more observation points ...
- Most of these data will never be seen by a human!
- Need for near-real time analysis of data feeds.
- Monitoring, intrusion, anomalous activity Classification, Prediction, Complex correlations, Detect outliers, extreme events, etc

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 Objects
 Objects

Continuous flow of data generated at **high-speed** in **Dynamic**, **Time-changing** environments.

The usual approaches for *querying*, *clustering* and *prediction* use **batch procedures** cannot cope with this streaming setting. Machine Learning algorithms assume:

• Instances are independent and generated at random according to some probability distribution \mathcal{D} .

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• It is required that $\mathcal D$ is stationary

Practice: *finite* training sets, *static* models.

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Data Streams

We need to maintain **Decision models** in **real time**. Decision Models must be capable of:

- incorporating new information at the speed data arrives;
- **detecting** changes and **adapting** the decision models to the most recent information.
- forgetting outdated information;

Unbounded training sets, dynamic models.

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Data Streams Models

Continuous flow of data generated at **high-speed** in **Dynamic**, **Time-changing** environments.

The input elements $a_1, a_2, \ldots, a_j, \ldots$ arrive sequentially, and describe an underlying function A:

- Insert Only Model: once an element a_i is seen, it can not be changed;
- **2** Insert-Delete Model: elements a_i can be deleted or updated.

The domain of variables can be huge.

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DBMS / DSMS

Data Base Management Systems

- Persistent relations
- One-time queries
- Random access
- Access plan determined by query processor and physical DB design

Data Streams Management Systems

- Transient streams (and persistent relations)
- Continuous queries
- Sequential access
- Unpredictable data characteristics and arrival patterns

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Traditional / Stream Processing

	Traditional	Stream
Nr. of Passes	Multiple	Single
Processing Time	Unlimited	Restricted
Memory Usage	Unlimited	Restricted
Type of Result	Accurate	Approximate
Distributed	No	Yes

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Puzzle: the missing number

Puzzle: Finding Missing Numbers

- Let π be a permutation of $\{1, \ldots, n\}$.
- Let π_{-1} be π with one element missing.
- $\pi_{-1}[i]$ arrives in increasing order

Task: Determine the missing number

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Puzzle: Finding Missing Numbers

- Let π be a permutation of $\{1, \ldots, n\}$.
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Task: Determine the missing number

Naive Solution:

Use a n-bit vector to memorize all the numbers (O(n) space)

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Puzzle: Finding Missing Numbers

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- Let π_{-1} be π with one element missing.
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Task: Determine the missing number

Streaming Solution: Use O(log(n)) space, or less

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Puzzle: Finding Missing Numbers

- Let π be a permutation of $\{1, \ldots, n\}$.
- Let π_{-1} be π with one element missing.
- $\pi_{-1}[i]$ arrives in increasing order

Task: Determine the missing number

Solution:

- Maintain $s = \sum_{1}^{i} (\pi_{-1}[i])$
- Return: $n \times (n+1)/2 \sum_{1}^{i} (\pi_{-1}[i])$

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Massive Data Sets

- Data analysis is **complex**, **interactive**, and **exploratory** over very large volumes of historic data.
- Traditional pattern discovery process **requires on-line ad-hoc queries**, not previously defined, that are successively refined.
- Due to the exploratory nature of these queries, an exact answer may not be required. A user may prefer a **fast approximate answer**.

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Massive Data Sets



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Approximate Answers

Approximate answers:

Actual answer is within 5 ± 1 with probability ≥ 0.9 .

• Approximation: find an answer correct within some factor

- Find an answer that is within 10% of correct result
- More generally, a $(1\pm\epsilon)$ factor approximation
- Randomization: allow a small probability of failure
 - Answer is correct, except with probability 1 in 1000
 - More generally, success probability $(1-\delta)$
- Approximation and Randomization: (ϵ, δ) -approximations The constants ϵ and δ have great influence in the space used. Typically the space is $O(1/\epsilon^2 \log(1/\delta))$.

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An Illustrative Example: Count-Min Sketch

Cormode & Muthukrishnan. An improved data stream summary: The count-min sketch and its applications. Journal of Algorithms, 2005. Used to approximately solve: Point Queries, Range Queries, Inner Product queries.

Simple sketch idea

- Creates a small summary as an array of $w \times d$ in size $W = 2/\epsilon$, $d = log(1/\delta)$
- Use d hash functions to map vector entries to [1..w]
- Works on Insert-only and Insert-Delete model streams

d



 $W=2/\epsilon,~d=\log(1/\delta)$

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Count-Min Sketch

Example: Count the number of packets from the set of IPs that cross a server in a network.



CM Sketch Update

Update:

Each entry in vector x is mapped to one cell per row. Increment the corresponding counter: $CM[k, h_k(j)] + = 1$.



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Count-Min Sketch

Example: Count the number of packets from the set of IPs that cross a server in a network.

CM Sketch Query

Query: How many packets from IP j? Estimate $\hat{x}[j]$ by taking $min_k CM[k, h_k(j)]$



return the minimum of values in blue cells

The estimate guarantees:

- $x[j] \leq \hat{x}[j]$
- $\hat{x}_i \leq \epsilon imes ||x_i||_1$, with probability $1 \delta_i$.

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Time Windows

- Instead of computing statistics over all the stream ...
- use only the most recent data points.
- Most recent data is more relevant than older data
- Several Window Models: Landmark, Sliding, Tilted Windows.
 - time based
 - sequence based



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Landmark Windows

• The recursive version of the sample mean:

$$\bar{x}_i = \frac{(i-1) \times \bar{x}_{i-1} + x_i}{i} \tag{1}$$

• Incremental version of the standard deviation:

$$\sigma_i = \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^2}{i}}{i - 1}}$$
(2)

• Recursive correlation coefficient.

$$corr(x,y) = \frac{\sum (x_i \times y_i) - \frac{\sum x_i \times \sum y_i}{n}}{\sqrt{\sum x_i^2 - \frac{\sum x_i^2}{n}} \sqrt{\sum y_i^2 - \frac{\sum y_i^2}{n}}}$$
(3)

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Sliding Windows

Computing these statistics over sliding windows: requires to maintain all the observations inside the window.

• Simple Moving Average:

$$SMA_t = \frac{(x_t + x_{t-1} + \ldots + x_{t-(n-1)})}{n}$$

where n is the window size

 Weighted moving average use multiplicative factors to give different weights to different data points

$$WMA_{t} = \frac{nx_{t} + (n-1)x_{t-1} + \dots + 2x_{t-n+2} + x_{t-n+1}}{n + (n-1) + \dots + 2 + 1}$$

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Sliding Windows

Maintaining Stream Statistics over Sliding Windows, M.Datar, A.Gionis, P.Indyk, R.Motwani; ACM-SIAM;2002 The basic idea:

• Use buckets of exponentially growing size $(2^0, 2^1, 2^2 \dots 2^h)$ to hold the data

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- Each bucket has a time-stamp associated with it
- It is used to decide when the bucket is out of the window

Data Structures for Exponential Histograms:

- Buckets: counts and time stamp
- LAST: stores the size of the last bucket.
- TOTAL: keeps the total size of the buckets.

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Exponential Histograms

When a new data element arrives:

- Create a new bucket of size 1 with the current time-stamp, and increment the counter TOTAL.
- Given a relative error, ε, if there are |1/ε|/2 + 2 buckets of the same size, merge the oldest two of the same-size into a single bucket of double size.
- The larger time-stamp of the two buckets is then used as the time-stamp of the newly created bucket.

Whenever we want to estimate the moving sum:

- Check if the oldest bucket is within the sliding window.
- If not, we drop that bucket and subtract its size from the variable TOTAL
- Repeat the procedure until all the buckets with timestamps outside of the sliding window are dropped.
- The estimate of 1's in the sliding window is TOTAL-LAST/2.

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Exponential Histograms: Example

Count the number of 1's in a sliding window

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$;
- Merge if |1/0.5|/2 + 2 = 3 buckets of the same size.

Basic Methods

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Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.



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Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.



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Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.



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Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.



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Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.



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Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.



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Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.



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Exponential Histograms: Example

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.

 Time = 8, x = 1

 EH: 2_2 2_4 1_6 1_8

 TOTAL: 6

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Exponential Histograms: Example

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.

Time = 9, x = 1 EH: 2_2 2_4 1_6 1_8 1_9 TOTAL: 7 Merge EH: 4_4 2_8 1_9 TOTAL: 7

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Exponential Histograms: Example

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.

Time = 10, x = 1

EH: 4₄ 2₈ 1₉ 1₁₀ TOTAL: 8

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Exponential Histograms: Example

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.

Time = 11, x = 1

EH: 44 28 210 111 TOTAL: 9

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Exponential Histograms: Example

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.

Time = 12, x = 1

EH: 44 28 210 111 112 TOTAL: 10

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Exponential Histograms: Example

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.

Time = 13, x = 1

EH: 4₄ 4₁₀ 2₁₂ 1₁₃ TOTAL: 11

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Exponential Histograms: Example

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.

Time = 14, x = 1

EH: 44 410 212 113 114 TOTAL: 12

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Exponential Histograms: Example

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Element	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0

- Window length=10;
- Relative Error $\epsilon = 0.5$ Merge if 3 buckets of the same size.

$\mathsf{Time} = 15, \, \mathsf{x} = \mathsf{0}$

TOTAL: 12

Removing outdated buckets

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	Timo	Data	Buckots	Total	Lact		
		Data	DUCKELS	TOLAI	LdSL		
		1	11	1	1		
	12	1	$1_1, 1_2$	2	1		
	Т3	1	$1_1, 1_2, 1_3$	3	1		
	(merge)		$2_2, 1_3$	3	2		
	Τ4	1	$2_2, 1_3, 1_4$	4	2		
	T5	0	$2_2, 1_3, 1_4$	4	2		
	Т6	1	$2_2, 1_3, 1_4, 1_6$	5	2		
			$2_2, 2_4, 1_6$	5	2		
	Τ7	0	$2_2, 2_4, 1_6$	5	2		
	Т8	1	$2_2, 2_4, 1_6, 1_8$	6	2		
	Т9	1	$2_2, 2_4, 1_6, 1_8, 1_8$.9 7	2		
			$4_4, 2_8, 1_9$	7	4		
	T10	1	$4_4, 2_8, 1_9, 1_{10} \\$	8	4		
	T11	1	$4_4, 2_8, 2_{10}, 1_{11} \\$	9	4		
	T12	1	$4_4, 2_8, 2_{10}, 1_{11} \\$, 1 ₁₂ 10	4		
	T13	1	$4_4, 4_{10}, 2_{12}, 1_{13}$	3 11	4		
	T14	1	$4_4, 4_{10}, 2_{12}, 1_{13}$	$_{3}, 1_{14}$ 12	4		
	(Removing	; outdat	ed)				
	Ť15	0	$4_{10}^{'}, 2_{12}^{'}, 1_{13}^{'}, 1_{13}^{'}$	14 8	4		

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Exponential Histograms: Analysis

- The size of the buckets grows exponentially: $2^0, 2^1, 2^2 \dots 2^h$
- Need only O(logN) buckets.
- It is shown that, for N 1's in the sliding window, we only need $O(logN/\epsilon)$ buckets to maintain the moving sum.
- The error in the oldest bucket **only**.
- The moving sum is proven to be bounded within the given relative error, ϵ .

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Sampling				

To obtain an unbiased sampling of the data, we need to know the length of the stream.

In Data Streams, we need to modify the approach!

When and How often should we sample?

Strategy

- Sample instances at periodic time intervals
- Useful to *slow down* data.
- Involves loss of information.

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Sampling				

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When and How often should we sample?

Strategy

- Sample instances at periodic time intervals
- Useful to *slow down* data.
- Involves loss of information.

Methods

- Reservoir Sampling, Vitter, 1985
- Min-Wise Sampling, Broder, et al., 00

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The reservoir Sample Technique

Vitter, J.; Random Sampling with a Reservoir, ACM, 1985.

- Creates uniform sample of fixed size k;
- Insert first k elements into sample
- Then insert *i*th element with prob. $p_i = k/i$
- Delete an instance at random.

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Analysis

Analyze the simplest case: sample size m = 1

Probability i'th item is the sample from a stream length n:

$$\frac{1}{2} \times \frac{2}{3} \dots \times \frac{i}{i+1} \times \dots \times \frac{n-2}{n-1} \times \frac{n-1}{n}$$

$$= 1/n$$

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Analysis

Known Problems

Low probability of detecting:

- Changes
- Anomalies

Hard to parallelize

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Outline

Motivation

- 2 Data Streams
 - Approximate Answers
 - Count-Min Sketch

3 Basic Methods

- Estimating statistics over windows
- Sampling
- Illustrative Applications
 - Toy Problem
 - Hot-Lists
 - Distributed Streams

5 References

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Illustrative Problem I

• Consider a high-speed stream of 0's and 1's



• Continuous query problem: Have we seen more 0s or 1s?

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Illustrative Problem II

A (real) data warehouse problem

- Suppose you have a retail data warehouse
- 3 TB of data
- 100s GB new sales records updated daily
- Millions of different items

Problem: hot-list

Identify *hot items*: the top-20 items in popularity Restricted memory: Can have a memory of 100s-1000s bytes only

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Illustrative Examples

- We see a large number of individual transactions.
 - What are the top sellers today?
- We are monitoring network traffic.
 - Which hosts/subnets are responsible for most of the traffic?
- We have a network of satellites monitoring events over large areas.
 - Which areas are experiencing the most activity over a week / day /hour?

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The Top-k Elements Problem

Count the top-K most frequent elements in a stream.

First Approach

Maintain a count for each element of the alphabet.

Return the k first elements in the sorted list of counts.

Problems

Exact and Efficient solution for small alphabets. Large alphabets: Space inefficient – large number of zero counts.

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The Space Saving Algorithm

Metwally, D. Agrawal, A. Abbadi, *Efficient Computation of Frequent and Top-k Elements in Data Streams*, ICDT 2005

Maintain partial information of interest; monitor only a subset m of elements.

- For each element *e* in the stream
 - If e is monitored: Increment Counte
 - Else
 - Let e_m be the element with least hits *min*.
 - Replace e_m with e with $count_e = min + 1$

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The Space Saving Algorithm: Properties

- Efficient for skewed data!
- Ensures no false negatives are kept in the top-k list: no non frequent item is in the top-k list.
- It allows false positive in the list: some non frequent items appear in the list.
- If the popular elements evolve over time, the elements that are growing more popular will gradually be pushed to the top of the list.



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Illustrative Problem III

Air Quality Monitoring

- Sensors monitoring the concentration of air pollutants.
- Each sensor holds a data vector comprising measured concentration of various pollutants (*CO*₂, *SO*₂, *O*₃, etc.).
- A function on the average data vector determines the Air Quality Index (AQI)
- Issue an alert in case the AQI exceeds a given threshold.

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Distributed Monitoring:

- Given:
 - A function over the average of the data vectors
 - A predetermined threshold
- Continuous Query: Alert when function crosses the threshold
- Goal: Minimize communication during query execution

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Example: Geometric Approach

I. Sharfman, A. Schuster, D. Keren, *A Geometric Approach to Monitoring Distributed DataStreams*, SIGMOD 2006

- Geometric Interpretation:
 - Each node holds a statistics vector
 - Coloring the vector space :
 - Grey: function > threshold
 - White: function ≤ threshold
- Goal: determine color of global data vector (average).



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Monitoring Threshold Functions



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The Bounding Theorem

- A reference point is known to all nodes
- Each vertex constructs a sphere
- Theorem: convex hull is bounded by the union of spheres
 - Local constraints!



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Basic Algorithm

- An initial estimate vector is calculated;
- Nodes compute spheres and check its color;
 - Drift vector is the diameter of the sphere
- If any sphere non monochromatic: node triggers re-calculation of estimate vector



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Monitoring Threshold Functions

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- Mostly Local Computations
- Minimum communications

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Software

Massive Online Analysis: http://moa.cms.waikato.ac.nz/



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Software				

scikit-multiflow is an open source machine learning framework for multi-output/multi-label and stream data. https://scikit-multiflow.github.io/

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scikit-multiflow

A machine learning framework for multi-output/multi-label and stream data.

C Github Repository



A Python package for online/streaming machine learning https://riverml.xyz/



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Resources

- Massive Data Analysis http://dimacs.rutgers.edu/~graham/
- Distributed Data Mining Maintained by Hillol Kargupta
- UCR Time-Series Data Sets Maintained by Eamonn Keogh, UCR, US http://www.cs.ucr.edu/~eamonn/time_series_data
- Mining Data Streams Bibliography Maintained by Mohamed Gaber http://www.csse.monash.edu.au/~mgaber/ WResources.html

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Data Stream Management Systems

- Niagara (OGI/Wisconsin) Internet XML databases
- Aurora (Brown/MIT) sensor monitoring, dataflow http://www-db.stanford.edu/sdt
- Stream (Stanford) general-purpose DSMS http://www-db.stanford.edu/stream/index.html
- COUGAR (Cornell)
- GigaScope and Hancock (At&T)
- Medusa (Brown University)

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- J. Gama, C.Pinto, *Discretization from data streams: applications to histograms and data mining.* SAC 2006.

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- Dynamically Maintaining Frequent Items Over A Data Stream, by C. Jin, W. Qian, C. Sha, J. Yu, A. Zhou; CIKM 2003.
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